The Maximum Electron Temperature
in the W VII-A Stellarator and
Comparison with Tokamak-Discharges

W VII-A Team

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Abstract

The maximum electron temperature of low density hydrogen discharges
in the W VII-A stellarator with ohmic heating is compared with
scaling laws derived by Guest, Miller, Pfeiffer, and Waltz.
The data show that the maximum temperature can be understood
on the basis of an electron heat conductivity χ_e which scales
with $\dfrac{I(r)}{r^2} \dfrac{1}{n_e \sqrt{T_e}}$ ($I(r)$ = plasma current within radius r). The
external rotational transform t_o affects the temperature by
determining the maximum current density in the plasma center
($j(o) \leq \dfrac{2}{\mu_o} \dfrac{B}{R}(1 - t_o)$, B = magnetic field, R = major radius).
An explicit dependence of χ_e on t_o cannot be derived from the
data. A comparison of the W VII-A data with tokamak data shows
that the scaling $T_e \sim \sqrt{Z_{eff} \, a^2 \dfrac{B}{R}(1-t_o) \, (1-r^2_{q=1})} \equiv \sqrt{Z}$ is a good
fit to the maximum electron temperature in ohmically heated
discharges rather than $T_e \sim \sqrt{\dfrac{BR^{3/2}}{a}} \sqrt{Z}$ as suggested by Guest
et al.. The condition for the applicability of this scaling
law is $(1-t_o)q_a \leq 4 - 5$, in this case the current density in
the plasma centre is determined by sawtooth oscillations.

Introduction

In a recent paper G.E. Guest et al. [1] have derived a semi-empirical formula for the electron thermal conductivity in Tokamak discharges. The authors suggest that the electron thermal conductivity is described by

$$\chi_e = \text{Const} \; \frac{a}{R^{3/2}B} \cdot \frac{I(r)}{r^2 n_e(r) \sqrt{T_e}} \tag{1}$$

R = major radius of the device, a = minor radius of the plasma, B = toroidal magnetic field, I(r) = plasma current within a cylinder with radius r, $n_e T_e$ = density, temperature of electrons.

A similar formula was derived before by Coppi et al. [2] in order to explain ALCATOR results. The factor $\frac{a}{R^{3/2}B}$ is derived by dimensional analysis according to a paper of Connor and Taylor [3]. For a given discharge $\frac{a}{R^{3/2}B}$ is a constant and does not affect the shape of the plasma profiles. The dependence on B can be checked in one experimental device but the existence of the factor $\frac{a}{R^{3/2}}$ can be proven only by comparison of different machines.

If the radiation losses and ion heating are neglected, the heat balance equation of the electrons can be solved explicitly, the temperature profile is proportional to $[1-(\frac{r}{a})^2]^2$ and the maximum electron temperature is given by the formula

$$T_e(o) = I \; [\frac{1}{2\pi^2} \; \frac{Z_{eff} \; \hat{\eta}}{f_o g(R,a)} \; \frac{\mu_o}{2\pi} \; \frac{R}{a^2}]^{1/2} \; [q_a \; \frac{q_a-1}{3}]^{1/2} \tag{2}$$

$$\hat{\eta} = \frac{e^2 c^2 \sqrt{m_e}}{1 \cdot 16} \; (\frac{\pi}{2})^{3/2} \quad , \; q_a = q \text{ on the boundary, } f_o = \text{const,}$$

$g = \frac{a}{R^{3/2}}$. This formula holds for $q_a \leq 4$. In this regime

a q=1 domain exists in the plasma centre and sawtooth
oscillations limit the current density to

$$J_{max}(o) = \frac{2B}{\mu_o R}$$

In a stellarator with ohmic heating the maximum current
density is given by $J_{max}(o) = \frac{2B}{\mu_o R}(1-t_o)$, where t_o is the
external rotational transform from the helical windings.

Also the radius of the q=1 surface is modified by the ex-
ternal transform. Following the procedure of ref.[1] the
electron energy balance equation can be solved taking into
account the external transform t_o. t_o is assumed to be
constant over the plasma radius, therefore this model
describes a shearless l=2 stellarator with ohmic heating.

The result of the calculation is

$$T_e(o) = Const \sqrt{\frac{B}{g(R,a)}} \; [a^2 Z_{eff} \frac{B}{R}(1-t_o)]^{1/2} \; [\frac{q_a - \frac{1}{1-t_o}}{3q_a}]^{1/2} \qquad (3)$$

The Constant const includes $\hat{\eta}$, f_e and numcerial factors.
In the case $t_o \to o$ formula (3) is reduced to formula (2).
The effect of the external transform is twofold: the maximum
current density in the centre is reduced by the factor $1-t_o$.
The radius of the q=1 surface is increased which also leads
to a reduction of the central temperature. With increasing
plasma density radiation losses [4] and heating of the ions
[5] play a dominant role in determining the maximum electron
temperature, therefore formula (3) yields an upper limit for
the temperature. In tokamak devices most data are taken
around q_a = 4, in this regime the effect of sawtooth os-
cillations is not yet important. If we neglect the reduction
of the effective plasma radius by sawtooth oscillations, the
maximum temperature is

$$T_e(o) \leq Const \; \frac{B}{g(R,a)} \; [a^2 Z_{eff} \frac{B}{R}(1-t_o)]^{1/2} \qquad (4)$$

Experimental Results

The formula (4) predicts the maximum temperature being in-
dependent of the plasma current. This effect is shown in
3 helium discharges in W VII-A with B = 3.3 T, t_0 = 0.17
(Fig. 1). The current is varied between 13 and 34 kA,
$n_e(o)$ = 5 x 10^{13}cm^{-3} in all three cases. The q=1 radius
as calculated from the temperature profiles (Z_{eff}=const)
expands from $r_{q=1}$ = 2 cm to $r_{q=1}$ = 6 cm (Fig. 2). The
central temperature even decays at high current indicating
the effect of the shrinking confinement zone. At 34 kA
q(a) ≃ 2 and the region of sawtooth oscillation extends
to more than 50% of the plasma radius. This was also con-
firmed by measurements of the X-ray diodes (Fig. 3).

The W VII-A data are taken from the hydrogen discharges
[6],[7], the density regime is $n_e(o)$ = 1 - 5 x 10^{13}cm^{-3}.
In this regime radiation losses or heating of ions are not
dominating. The maximum temperature is plotted versus
$Z =: Z_{eff} \frac{B}{R}(1-t_0) (\frac{a^2}{0.11})[\frac{4}{3}(1-\frac{t_p}{1-t_0})]$, the magnetic field was varied
between 2.5 T and 3.5 T, t_0 = 0.055 - 0.23. The case t_0 = 0.055
is comparable with tokamak discharges since the transform t_0
is much smaller than the transform of the heating current.

The data of W VII-A show a linear increase of the central

temperature $T_e(o)$ with $\sqrt{Z_{eff} \frac{B}{R} (1-t_0)[\frac{4}{3} (1 - \frac{t_p(a)}{1-t_0}]}$ (Fig.5)

the optimum can roughly be described by

$$T_{e,max} = 360 \sqrt{Z_{eff} \frac{B[T]}{R[m]}(1-t_0) [\frac{4}{3}(1-\frac{t_p(a)}{1-t_0})]} \text{ [eV]},$$

the bulk of the data follows the line $T_e = 290 \sqrt{Z}$ [eV]

The maximum deviation is 25 % which might be caused by the neglected effects: radiation, ion heating and the shrinkage of the confinement zone due to changes of the profile.

Therefore, the conclusion can be drawn that the maximum temperature in W VII-A roughly scales with $Z_{eff} \frac{B}{R}(1-t_o)$, whether the factor $\sqrt{B}$ (see eq. (4)) exists or not cannot be decided since the variation in B is too small. It also cannot be decided whether the thermal conductivity χ_e explicitly depends on the external helical field. If there is any dependence of χ_e on t_o, this dependence must be small, for $t_o \leq 0.23$ the variation of χ_e with t_o cannot be more than 50%, otherwise the maximum temperature would be outside the experimental data. The total transform $t_p + t_o$ has been varied only by a factor of 2, therefore a strong variation of the confinement with t_o cannot be expected.

The radius of the q=1 surface is derived neglecting the effect of radiation on the temperature and current profiles. A shrinkage of the current channel due to increasing radiation leads to a steepening of the current profile and an increase of the q=1 regime. Therefore, the reduction factor $\frac{4}{3}(1- \frac{t_p}{1-t_o})$ calculated above is too large, sawtooth oscillations and a q=1 surface already occurs for $\frac{t_p}{1-t_o} < \frac{1}{4}$ in many devices. In the calculations above the effect of a radiation layer and the shrinkage of the current channel can be simulated by a reduced plasma radius $a \rightarrow a(1-\Delta)$, Δ is the width of the radiation layer.

Fig. 4 shows the width of the confinement layer $a - r_{q=1}$ vs. $\frac{t_p(a)}{1-t_o}$, the experimental data in Fig. 4 are taken from helium discharges in W VII-A. It can be seen that especially at low plasma currents the radius $r_{q=1}$ is larger than calculated in this simple model.

Comparison With Tokamak Discharges

In order to check equation (3) data are collected from
several tokamak experiments (PULSATOR, ALCATOR, TFR, ST,
PLT, Frascati Tokamak, T-10). Most of the data are taken
from a collection by Pfeiffer and Waltz [8] and from the
results reported at the Berchtesgaden and Innsbruck
Conferences [9] ,[10], [11], [12], [13].

The maximum temperature is plotted versus Z, the factor
$\sqrt{\frac{B}{g(R,a)}}$ is not taken into account in Fig. 6. Since the
tokamak data are mainly taken at $q(a) \geq 4$ ($t_p \leq 0.25$) only
some PULSATOR data have $q(a) = 3.5$, the reduction factor due
to sawtooth oscillations is set to 1 in eq. (3). The data
also include the high density discharges, therefore it has
to be expected that especially in this case $T_e < 360 \sqrt{Z}$ [eV].

Fig. 6 shows that the maximum temperature in the ohmically
heated devices can be described by $360 \sqrt{Z}$ [eV]. Close to the
high density limit (PULSATOR, ALCATOR), the deviation from
this line is maximum. In order to check the existence of
the factor $\sqrt{\frac{B}{g(R,a)}}$, the normalized temperature $T_e \sqrt{\frac{g(R,a)}{B}}$
is plotted versus Z (Fig. 7).

A comparison of Fig. 6 with Fig. 7 shows that the scaling
$T_e \sim \sqrt{Z}$ is a better fit to the experimental data than
$\sqrt{\frac{B}{g(R,a)}} \cdot \sqrt{Z}$. The difference between the two figures is not
very large, but there are some facts which are in favour of
the $T_e \sim \sqrt{Z}$ scaling. In Fig. 7 the ISX data are above the
optimum line of the other tokamak data, but as it was pointed
out in the ISX paper [5] the heating of ions already plays an
important role in the ISX device. Therefore, it should be
concluded that either ISX and W VII-A follow a scaling of the
thermal conductivity of the electrons different from other
tokamaks, or - as shown in Fig. 6 - the ohmically heated
discharge is described by $T_e \sim \sqrt{Z}$. But this means that the
factor $\sqrt{\frac{B}{g(R,a)}}$ does not exist.

Discussion

The scaling law $T_e(o) \sim \sqrt{Z}$ as confirmed by the experimental data does not necessarily confirm the radial dependence of the thermal conductivity $\chi_e \sim \dfrac{I(r)}{r^2 n \sqrt{T_e}}$. Any function $\chi_e(r) \sim \dfrac{I(r)}{r^2} \dfrac{g(r)}{n\sqrt{T_e}}$, where $g(r)$ is a normalized function $\int_o^r g(r)\, dr = 1$ describing the localization of fluctuations which give rise to the anomalous conductivity, can lead to the same scaling laws (within a constant factor). If $g(r)$ is independent of the plasma current, temperature and density, the scaling law of $T_e(o)$ is also independent of $g(r)$.

Acknowledgements

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Figure Captions
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Fig. 1: Electron temperature profile of
three helium discharges with different
current (B = 3.3 T, ℓ_o = 0.17).

Fig. 2: Profile of the rotational transform (ℓ(r)
under the same conditions as in Fig. 1.

Fig. 3: Intensity of sawtooth oscillations measured
by X-ray diodes. Comparison with the
electron temperature profile.

Fig. 4: Radius of the confinement zone a-r$_{q=1}$ as
a function of $\dfrac{\ell_p(a)}{1-\ell_o}$. The experimental data
are derived from the $T_e(r)$ profiles and the
X-ray measurements (see Fig. 3).

Fig. 5: Maximum electron temperature T_e(o) vs.
$Z_{eff} \dfrac{B}{R} (1-\ell_o) \dfrac{4}{3} (1 - \dfrac{\ell_p(a)}{1-\ell_o})$. data are from
hydrogen discharges in W VII-A.

Fig. 6: Maximum electron temperature T_e(o) vs.
$(\dfrac{a}{0.11})^2 Z_{eff} \dfrac{B}{R}(1-\ell_o) \dfrac{4}{3}(1-\dfrac{\ell_p(a)}{1-\ell_o})^e$ for second
ohmically heated discharges. Since most of
the tokamak data are from the regime q_a = 4 - 6,
the reduction factor $\dfrac{4}{3}(1 - \dfrac{\ell_p(a)}{1-\ell_o}$ is set to 1 in
this case.

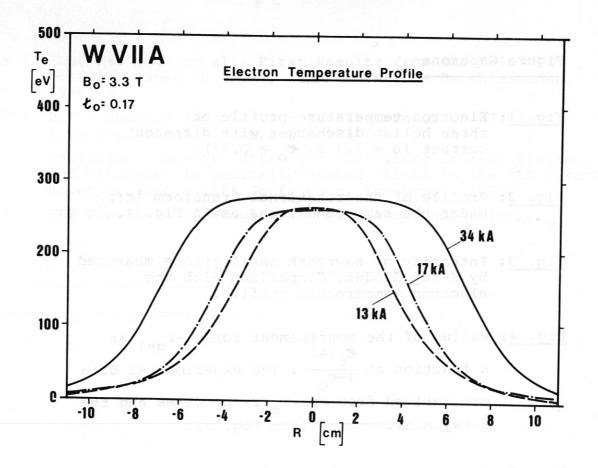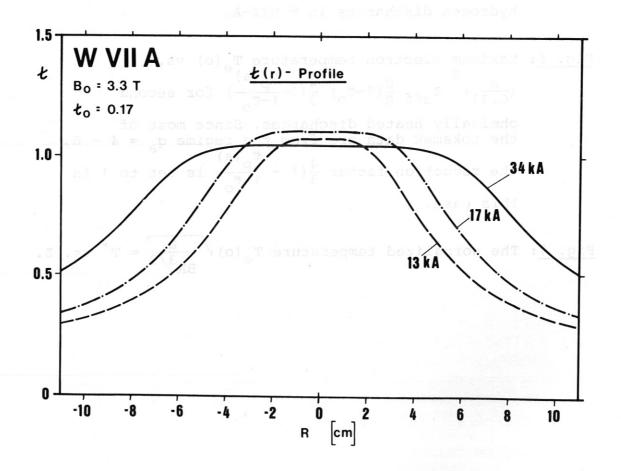Fig. 7: The normalized temperature $T_e(o)\sqrt{\dfrac{a}{BR^{3/2}}} = T^*$ vs. Z.

Fig. 1

Fig. 2

Fig. 3

W VII A

$B_0 = 3.5$ T

$t_0 = 0.14$

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Fig. 4

W VII A

a = 11 cm

● = from T_e-profile

○ = from X-ray-measurements

Radius of the q = 1 - surface

$r_q = 1$

q = 1 - region

$\Delta = 0$

$\Delta = 0.1$

$\Delta = 0.2$

$\Delta = 0.3$

$(1 - \frac{r}{a})$

$\frac{t_p(a)}{1 - t_0}$

Fig. 5

Maximum Electron Temperature

$T_e(o)$
eV

W VIIA

H_2

○ $t_o = 0.23$
● $t_o = 0.11$
▲ $t_o = 0.055$

$T_e(o) = 360\sqrt{Z}$

$T_e = 290\sqrt{Z}$

$Z = Z_{eff} \dfrac{B(T)}{R\,[m]} (1-t_o) \dfrac{4}{3} \left(1 - \dfrac{t_p(a)}{1-t_o}\right) \left(\dfrac{a\,[m]}{0.11}\right)^2$

Fig. 6

Maximum Electron Temperature
of Ohmically Heated Discharges.

Fig. 7